

A Brief Summary of Theoretical Backgrounds in VORTEX-Winds On-line Wind Simulator (VOWS)

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1. Modelling and simulation of wind fields

The wind velocity fields simulated by NOWS are modelled as a stationary, Gaussian random processes depending on time t and of the position $\mathbf{x} = [x, y, z]^T$ in a finite spatial domain D . The orthogonal reference system (x, y, z) has origin on the ground and the axis z is vertical and directed upwards.

The wind velocity is idealized as the sum of two terms: (1) the mean wind velocity $U(\mathbf{x})$, function of the position \mathbf{x} , and oriented in the whole domain along the positive direction of the axis x ; (2) the zero-mean random function $\mathbf{u}(\mathbf{x}, t)$, function of space and time, whose Cartesian components u, v, w along x, y , and z , respectively, represent the longitudinal, lateral and vertical atmospheric turbulence components.

NOWS can produce realizations of the wind velocity field in any user-defined discrete set of points in D , which is referred to as simulation domain.

2. Models for mean wind velocity

The mean wind velocity is aligned along the x -axis in the whole simulation domain. Its amplitude in the simulation points can be assigned through three alternative options:

1. the mean wind velocity depends on z according to the logarithmic profile:

$$U(z) = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0}\right) \quad \text{for } z > z_{\min} \quad (1)$$
$$U(z) = U(z_{\min}) \quad \text{for } z \leq z_{\min}$$

where κ is the Von Kármán constant (it is assumed $\kappa = 0.4$), z_0 is the roughness length and u_* is the friction velocity. The profile defined by Eq. (1) is usually accepted for $z < 200$ m and is assumed constant for $z < z_{\min}$; the friction velocity u_* is a scale parameter, while z_0 determines the shape of the profile according to the superficial characteristics of the terrain. Typical values for z_0 and z_{\min} can be found in Table 1 [1].

2. The mean wind velocity follows the logarithmic profile [1]:

$$U(z) = k_r v_b \ln\left(\frac{z}{z_0}\right) \quad \text{for } z > z_{\min} \quad (2)$$
$$U(z) = U(z_{\min}) \quad \text{for } z \leq z_{\min}$$

where v_b is referred to as basic wind velocity and represents the wind velocity at the reference height ($z = 10$ m) on the reference terrain (category II, Tab. 1) with an assigned return period [1]; k_r is referred to as terrain factor and is defined as a function of z_0 through the relationship:

$$k_r = 0.19 \left(\frac{z_0}{z_{0,II}}\right)^{0.07} \quad (3)$$

where $z_{0,II} = 0.05$ m is the roughness length pertinent to the reference terrain (Tab. 1).

3. The mean wind velocity is assigned in each simulation point through an input vector.

Table 1. Terrain categories defined in [1].

Terrain category	z_0 (m)	z_{\min} (m)
0 Sea or coastal area exposed to the open sea	0.003	1
I Lakes or flat and horizontal area with negligible vegetation and without obstacles	0.01	1
II Area with low vegetation such as grass and isolated obstacles (trees, buildings) with separations of at least 20 obstacle heights	0.05	2
III Area with regular cover of vegetation or buildings or with isolated obstacles with separations of maximum 20 obstacle heights (such as villages, suburban terrain, permanent forest)	0.3	5
IV Area in which at least 15 % of the surface is covered with buildings and their average height exceeds 15 m	1.0	10

3. Models for turbulence

The fluctuating component of the wind velocity is modelled as a Gaussian, zero-mean, stationary random process. The three components of turbulence are considered as statistically-independent and are simulated independently of each other.

The Power Spectral Density (PSD) function of the turbulence component $\varepsilon = u, v, w$ in the point \mathbf{x} of D is defined as [2]:

$$S_{\varepsilon}(\mathbf{x}, n) = \sigma_{\varepsilon}^2(\mathbf{x}) \frac{d_{\varepsilon} \frac{L_{\varepsilon}(\mathbf{x})}{U(\mathbf{x})}}{\left(1 + 1.5d_{\varepsilon} n \frac{L_{\varepsilon}(\mathbf{x})}{U(\mathbf{x})}\right)^{5/3}} \quad (\varepsilon = u, v, w) \quad (4)$$

where n is the frequency, d_{ε} is a constant defined as $d_u = 6.868$ and $d_v = d_w = 9.434$; L_{ε} and σ_{ε} are, respectively, the integral length scale and the standard deviation of turbulence component $\varepsilon = u, v, w$. These quantities can be assigned in each simulation point through an input vector or can be defined through the models [1, 2]:

$$L_{\varepsilon} = 300\lambda_{\varepsilon} \left(\frac{z}{200}\right)^{\nu} \quad (\varepsilon = u, v, w) \quad (5)$$

$$\sigma_{\varepsilon}^2 = \beta_{\varepsilon} u_*^2$$

where $\lambda_u = 1.00$, $\lambda_v = 0.25$, $\lambda_w = 0.10$ and

$$\beta_u = 6 - 1.1 \arctan(\ln z_0 + 1.75)$$

$$\beta_v = 0.55\beta_u \quad (6)$$

$$\beta_w = 0.25\beta_u$$

$$\nu = 0.67 + 0.05 \ln z_0 \quad (7)$$

In Eqs. (5), (6) and (7) the dimensional quantities L_{ε} , z and z_0 are expressed in metres.

The two-point PSD of the turbulence component ε in the points \mathbf{x} and \mathbf{x}' of D is expressed as:

$$S_{\varepsilon\varepsilon}(\mathbf{x}, \mathbf{x}', n) = \sqrt{S_{\varepsilon}(\mathbf{x}, n) S_{\varepsilon}(\mathbf{x}', n)} \text{Coh}_{\varepsilon\varepsilon}(\mathbf{x}, \mathbf{x}', n) \quad (8)$$

where $\text{Coh}_{\varepsilon\varepsilon}$ is the two-point coherence function of the turbulence component ε defined as [3]:

$$\text{Coh}_{\varepsilon\varepsilon}(\mathbf{x}, \mathbf{x}', n) = \exp\left(-\frac{2n\sqrt{C_{x\varepsilon}^2 |x - x'|^2 + C_{y\varepsilon}^2 |y - y'|^2 + C_{z\varepsilon}^2 |z - z'|^2}}{U(\mathbf{x}) + U(\mathbf{x}')}\right) \quad (9)$$

where $C_{x\varepsilon}$, $C_{y\varepsilon}$ and $C_{z\varepsilon}$ are the exponential decay coefficients for the turbulence component ε and separation distance along the direction x , y , and z , respectively. The value for these parameters is assumed as [2]:

$$\begin{aligned} C_{xu} &= 3.0 & C_{xv} &= 3.0 & C_{xw} &= 0.5 \\ C_{yu} &= 10.0 & C_{yv} &= 6.5 & C_{yw} &= 6.5 \\ C_{zu} &= 10.0 & C_{zv} &= 6.5 & C_{zw} &= 3.0 \end{aligned} \quad (10)$$

4. Simulation algorithm

The simulation of the turbulence field is carried out by the spectral-representation algorithm described in [4]. The procedure is organized as follows:

1. The PSD matrix \mathbf{S} of the turbulence component ε selected for the simulation is obtained evaluating Eqs. (8) for every pair of simulation points \mathbf{x}_j and \mathbf{x}_k ($j, k = 1, \dots, N$) in the simulation domain

$$[\mathbf{S}(n)]_{jk} = S_{\varepsilon\varepsilon}(\mathbf{x}_j, \mathbf{x}_k, n) \quad (j, k = 1, \dots, N) \quad (11)$$

2. The frequency n is discretised according to a linear or logarithmic scale. The linear scale is defined as:

$$n_h = h\Delta n \quad (h = 1, \dots, N_n) \quad (12)$$

in which N_n is given by the relationship

$$N_n = \text{ceil}((N_t - 1)/2) \quad (13)$$

where $\text{ceil}(\bullet)$ returns the integer number immediately greater than the argument and N_t is the number of time steps required in the simulation.

The logarithmic scale is defined as:

$$n_h = \frac{1}{N_t \Delta_t} \left(\frac{N_t}{2} \right)^{\frac{h-1}{N_n-1}} \quad (h = 1, \dots, N_n) \quad (14)$$

in which Δ_t is the simulation time step and N_n is user-defined.

3. The PSD matrix is factorized in terms of eigenvalues and eigenvectors for each frequency value defined by Eq. (12) or (14).

$$\mathbf{S}(n_h) = \mathbf{\Theta}(n_h)^T \mathbf{\Gamma}(n_h) \mathbf{\Theta}(n_h) \quad (h = 1, \dots, N_n) \quad (15)$$

where $\mathbf{\Gamma}$ and $\mathbf{\Theta}$ are, respectively the matrices containing the eigenvalues and eigenvector of \mathbf{S} . Since the factorization defined by Eq. (15) must be repeated N_n times, for large N_t and N , it is convenient to adopt the logarithmic discretisation scheme defined by Eq. (14) for which N_n is user-defined.

When the logarithmic frequency discretisation is selected NWS adopts $N_n = 50$.

4. Realizations of the turbulence field are generated by an FFT algorithm [4]. If the factorization of the matrix \mathbf{S} is carried out according to the logarithmic frequency scale, values of $\mathbf{\Gamma}$ and $\mathbf{\Theta}$ to be employed in the FFT are interpolated according to the schemes described in [4].

References

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